

Explosive Properties of Reactor-Grade Plutonium

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The following discussion focuses on the question of whether a terrorist organization or a threshold state could make use of plutonium recovered from light-water-reactor fuel to construct a nuclear explosive device having a significantly damaging yield. Questions persist in some nonproliferation policy circles as to whether a bomb could be made from reactor-grade plutonium of high burn-up, and if so, whether the task would be too difficult for a threshold state or terrorist group to consider. Although the information relevant to these questions is in the public domain, and has been for a considerable time, it is assembled here for use by policy makers and members of the public who are concerned about preventing the spread of nuclear explosives.

INTRODUCTION

Plutonium-239 is produced in nuclear reactors through neutron capture by U-238 and two successive β -decays. In addition to the isotope Pu-239, the plutonium extracted from reactor fuel will contain other plutonium isotopes formed as a result of successive neutron capture or $(n, 2n)$ reactions. At very low burn-up, the fractional amounts of the secondary isotopes are very small. For example, the fraction of Pu-240 may be a few percent of the total plutonium, with the fraction of Pu-241 being approximately an order of magnitude smaller, and that of Pu-242 an order of magnitude smaller still. Such plutonium is characteristic of that used for weapons.

In commercial reactors, burn-ups are much higher than in reactors dedicated to production of weapons plutonium, and at higher burn-ups the fractional amounts of the heavier isotopes increase, as shown in figure 1 for light-water reactors. At a burn-up of 33,000 MWd te^{-1} (characteristic of most pres-

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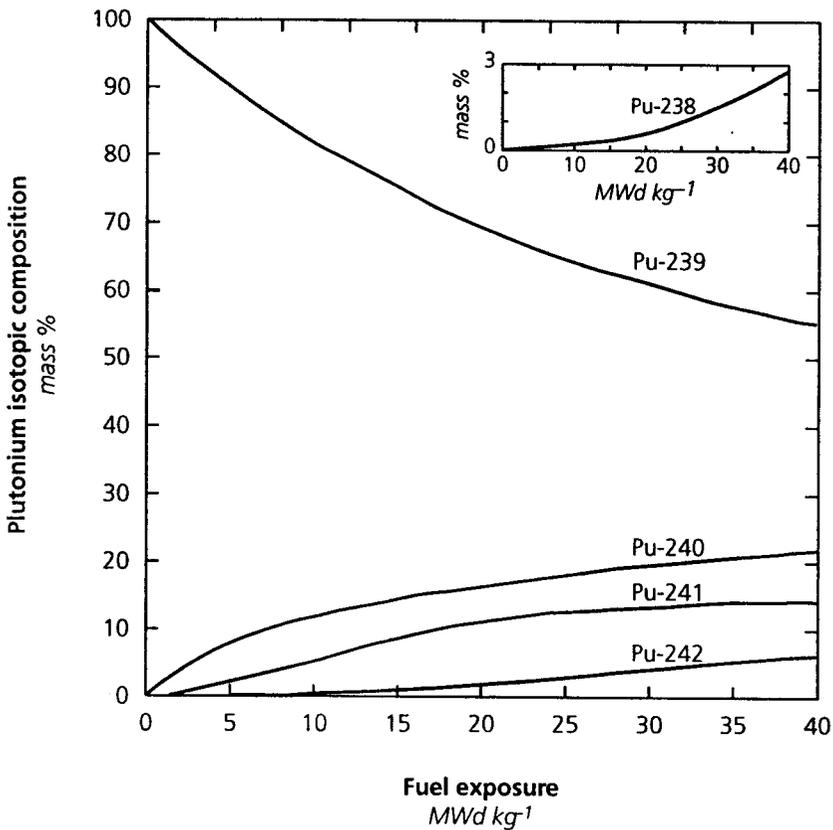


Figure 1: Plutonium isotope composition as a function of fuel exposure in a pressurized-water reactor, upon discharge.

surized-water-reactor spent fuel today), the fraction of plutonium isotopes upon discharge would typically be 59 percent Pu-239, 21 percent Pu-240, 14 percent Pu-241, and 5 percent Pu-242. Of the other plutonium isotopes that would also be present in relatively quite small amounts, the most prominent is Pu-238, which would reach a level of one or two percent. We consider in the following whether plutonium with relatively high fractions of Pu-240, Pu-241, and Pu-242 characteristic of plutonium recovered from commercial power reactors (i.e., “reactor-grade” plutonium) could be used in a nuclear explosive. What would be the effect of reactor-grade plutonium on the critical mass required for a nuclear explosion? What would be the probability of predetonation in such a mass and what would be its resulting “fizzle yield”? Table 1 shows the isotopic composition for various grades of plutonium.

Table 1: Approximate isotopic composition of various grades of plutonium.

Grade	Isotope				
	Pu-238	Pu-239	Pu-240	Pu-241 ^a	Pu-242
Super-grade	–	.98	.02	–	–
Weapons-grade ^b	.00012	.938	.058	.0035	.00022
Reactor-grade ^c	.013	.603	.243	.091	.050
MOX-grade ^d	.019	.404	.321	.178	.078
FBR blanket ^e	–	.96	.04	–	–

a. Pu-241 plus Am-241.
b. N.J. Nicholas, K.L. Coop and R.J. Estep, *Capability and Limitation Study of DDT Passive-Active Neutron Waste Assay Instrument* (Los Alamos: Los Alamos National Laboratory, LA-12237-MS, 1992).
c. Plutonium recovered from low-enriched uranium pressurized-water reactor fuel that has released 33 megawatt-days kg^{-1} fission energy and has been stored for 10 years prior to reprocessing (*Plutonium Fuel: An Assessment* (Paris: OECD/NEA, 1989), Table 12A).
d. Plutonium recovered from 3.64 percent fissile plutonium mixed-oxide (MOX, uranium-plutonium) MOX fuel produced from reactor-grade plutonium and which has released 33 megawatt-days kg^{-1} fission energy and has been stored for 10 years prior to reprocessing (*Plutonium Fuel: An Assessment* (Paris: OECD/NEA, 1989), Table 12A).
e. FBR = Fast-neutron plutonium Breeder Reactor.

CRITICALITY PROPERTIES OF REACTOR-GRADE PLUTONIUM

As shown in figure 2, which plots the neutron cross-section for fission against neutron energy for the principal plutonium and uranium isotopes (and americium-241, a decay product of Pu-241) all of the plutonium isotopes are fissionable. Indeed, a bare critical assembly could be made with plutonium metal no matter what its isotopic composition might be. The number of neutrons per fission (approximately three) is the same for Pu-239, Pu-240, Pu-241 and Pu-242. The odd isotopes (239 and 241) are both “fissile”—that is, fission may be induced in them by neutrons of any energy, whether slow or fast. Their fission cross-sections differ in detail but are similar enough that their bare critical masses* are nearly equal, being about 15 kilograms in δ -phase metal.[†]

For Pu-240, the fission threshold is close to one MeV, but above one MeV the fission cross-section, though smaller than that of Pu-239, is larger than that of U-235. The bare critical mass of Pu-240 in α -phase metal is about 40 kilograms. Since the bare critical mass of weapons-grade uranium (94 percent

* The bare critical mass (“bare crit”) of a material at standard density is the critical mass with no neutron reflector present.

† Plutonium metal can exist in six allotropic forms corresponding to six different crystalline configurations. The two forms most often mentioned with respect to weapons are the α -phase (density = 19.6 gm cm^{-3}) and the δ -phase (density = 15.7 gm cm^{-3}).

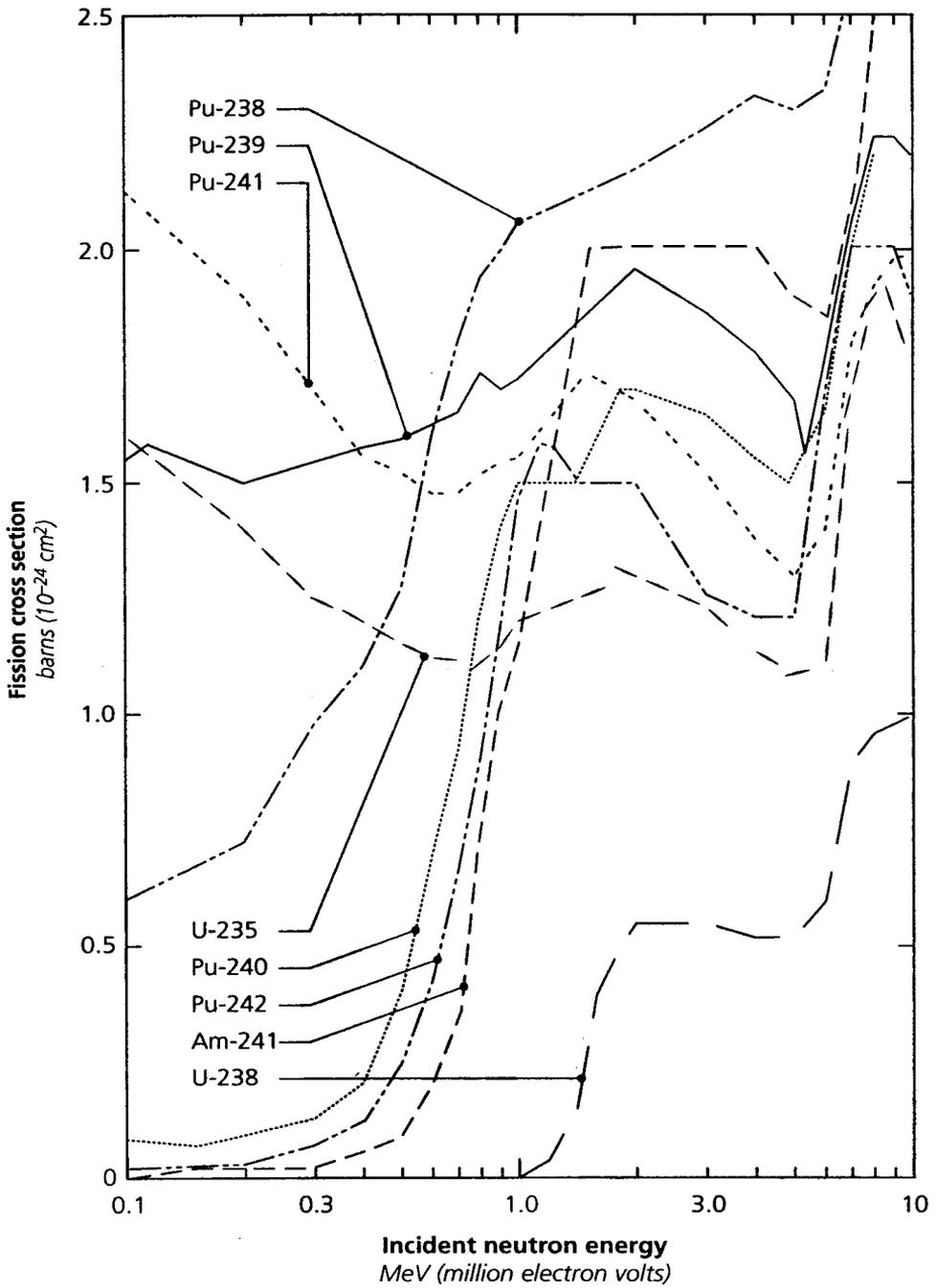


Figure 2: The neutron cross-section for fission of the principal plutonium and uranium isotopes (and americium-241, a decay product of Pu-241) against neutron energy.

Table 2: Various properties of plutonium isotopes (and americium-241).

Isotope	Half-life ^a	Bare critical mass	Spontaneous fission neutrons	Decay heat
	years	kg, α -phase	(gm-sec) ⁻¹	watts kg ⁻¹
Pu-238	87.7	10	$2.6 \cdot 10^3$	560
Pu-239	24,100	10	$22 \cdot 10^{-3}$	1.9
Pu-240	6,560	40	$0.91 \cdot 10^3$	6.8
Pu-241	14.4	10	$49 \cdot 10^{-3}$	4.2
Pu-242	376,000	100	$1.7 \cdot 10^3$	0.1
Am-241	430	100	1.2	114

a. By α -decay, except Pu-241, which is by β -decay to Am-241.

U-235) is 52 kilograms, Pu-240 may be said to be a more effective fissionable material than weapons-grade uranium in a metal system.

In considering the isotope Pu-242 it is convenient to consider also the isotope Am-241. This is the daughter of Pu-241, which decays by β -emission with a half-life of 14.7 years. Though not present in freshly separated reactor plutonium, it grows in gradually; and, because there is more Pu-241 than Pu-242 in spent fuel, the abundance of Am-241 could eventually exceed that of Pu-242. The fission characteristics of Am-241 are much less favorable than those of Pu-241, so that as the replacement proceeds the critical mass of the plutonium will increase with time after the separation of the plutonium, though not very rapidly and never to a very great extent. Each of the isotopes Pu-242 and Am-241 has a fission threshold close to one MeV. Above one MeV, their fission cross-sections are similar, each being larger than that of U-235.

In practice, at all burn-up levels and at any time following discharge the critical mass of reactor-grade plutonium metal is intermediate between that of Pu-239 and Pu-240, which is more reactive than weapons-grade uranium, reactor-grade plutonium can be brought to a supercritical—and, hence, explosive—state by any assembly system that can handle U-235. Table 2 shows the bare critical masses of the different plutonium isotopes.

The bare critical masses are not the masses one would need to construct a device, since by the use of a neutron reflector a few inches thick, the critical mass of each of these materials can be reduced by a factor of two, or so, below the bare critical mass. However, the relative ranking of the critical masses will be preserved very nearly as reflectors are applied.

THE CHAIN REACTION

A single neutron released in a plutonium system may, with various probabilities, induce a fission (from which three neutrons emerge), escape from the system, or disappear as a result of capture. (In a metal system the last probability is quite small, and may be ignored here.) Letting k denote the number of direct descendants of the original neutron that do not escape the system, the net change in the neutron population will be $(k - 1)$, and the rate of change will be $(k - 1) / \tau$ where τ is the mean lifetime of a neutron in the system. Setting $(k - 1) / \tau = \alpha$, the population of neutrons in a chain started by a single neutron at time zero will be $e^{\alpha t}$.

In a subcritical system, k is less than one, α is negative, and the population decreases exponentially. In a critical system $k = 1$, $\alpha = 0$, and the neutron population remains constant in time. At critical, then, the probability of a neutron causing a fission is one-third, and the probability of escape is two-thirds. In a supercritical system k is greater than one, $(k - 1)$ and α are positive, and the neutron population increases exponentially by a factor of e in each time interval of α^{-1} . Though the numerical range of $(k - 1)$ is quite limited (between zero and two—and only approaching two when no neutrons escape, that is, in an infinite medium), it does provide the whole measure of the effect of the degree of supercriticality on the exponential rate of growth of the chain reaction.

The neutron lifetime, τ , in a metal system is a very small number. The total mean track length of a neutron in uncompressed δ -phase plutonium metal from birth to subsequent fission is about 15 centimeters. (Because a collision with a nucleus results in scattering several times more frequently than it results in fission, this 15 centimeter track length usually consists of a number of shorter segments travelled in a nearly random selection of directions.) The average energy of a fission neutron moving in plutonium after a few scatterings is about one MeV, so its velocity is close to $1.4 \cdot 10^9$ cm sec⁻¹. Its lifetime, τ , is consequently close to 10^{-8} seconds, and α is close to $(k - 1) 10^8$ sec⁻¹. This value of α will, of course, vary directly with the density of the material because the track length (and hence τ) vary inversely with the density.

Near the start of a chain reaction, with only a few fissions per gram of material, there will be no effect on the state of the material. In fact, it requires about e^{35} fissions to provide one calorie per gram in a mass of about 10 kilograms of plutonium, and this will merely raise the temperature of the material by about 30°C, which will have no appreciable effect on the size or shape of the material or its condition of motion. However, with about e^{42} fis-

sions in a 10 kilogram system the energy provided by fission will be about one kilocalorie per gram, which is the energy typically released by the detonation of high explosives. By this stage in the chain reaction, the plutonium will have vaporized and begun to exert a pressure in the megabar range on its surroundings. Such pressures will override any residual forces involved in driving the assembly, and in a very short time will initiate a rapid expansion of the core. The establishment of this motion of disassembly may be thought of as the start of the explosion. One cannot attach a very precise value to this moment, nor is precision on this point of any importance. We shall consequently assign the value of e^{45} fissions as marking the start of the explosion.

If the chain reaction starts only after the assembly is complete, the value of α at the start of the explosion will be the nominal value, α_0 , associated with the completed assembly. If the chain starts well before the assembly is complete, while α is still rising towards α_0 , the value of α at the start of the explosion will be the value reached when the integral of $\alpha \cdot dt$ taken from the time of initiation of the chain equals 45. The smallest explosion resulting from preinitiation will be that resulting from a chain starting at the earliest possible time, which is just as the system becomes critical in the course of its assembly.

In all cases α will still be positive at the start of the explosion so that the neutron population and the rate of energy generation will continue to increase, even though the value of $(k - 1)$ (and hence α) will be decreasing as a result of the expansion and consequent reduction in supercriticality of the core. This will continue until $(k - 1)$ falls to zero (a moment that may be referred to as "second critical"), at which point the neutron population reaches its maximum, as also, almost simultaneously, does the energy generation rate. From this point on the system is subcritical, the neutron population falls rapidly, and though energy continues to be generated, it does so at a decreasing rate until all the neutrons have leaked away. A significant fraction of the total energy release will be generated during this subcritical phase of the disassembly.

It will be evident merely from consideration of the progress of a nuclear explosion that, for any particular system with core and neutron reflector specified, the smaller the degree of supercriticality (the smaller the value of α at explosion time), the smaller the final energy release. But this, by itself, does not enable one to assess the extent of the yield degradation associated with one or another reduction in the value of α at the start of the explosion. However, Robert Serber, in the *Los Alamos Primer*¹ presented a qualitative argument to show that the yield of a particular assembly would vary approximately with the value of α^3 at the start of the explosion. (Serber's

notation is quite different from that used here, and the approximations involved were applicable only to systems having a limited degree of supercriticality, but his conclusions, though qualitative, will be adequate for our needs, which are also qualitative.)

THE FIZZLE YIELD

As a purely hypothetical example we consider an assembly of the solid implosion type used at Trinity (the first U.S. nuclear test, 16 July 1945). We assume a core-reflector combination for which the critical mass is about one half a bare critical mass. The assembly must be subcritical as built, but, to obtain as favorable a performance as possible, we suppose the assembly is close to critical as built. A δ -phase plutonium core mass could, then, be in the neighborhood of seven or eight kilograms, and thus have a radius close to five centimeters. Since the ingoing shock wave from the high explosive would compress the reflector somewhat, the system would become critical at about the time the shock reached the core radius. Having a velocity close to five km sec⁻¹ the shock would transit the core in about 10⁻⁵ seconds. The time interval, t_0 , through which the system is supercritical prior to completion of the assembly as the shock reaches the center is, then, about 10⁻⁵ seconds. We shall further assume that in the final state ($k - 1$) is close to unity, that is, in the middle of the supercritical range, from zero to two. On this basis, the nominal value of α for this hypothetical system will be $\alpha_0 = 10^8$, and the quantity $\alpha_0 \cdot t_0$ will be approximately 10³. We denote the nominal yield of the explosion as Y_0 .

As a first rough approximation, we assume that α varies linearly with time, that is, $\alpha = c \cdot t$. (A similar approximation was also used by Serber. It is certainly not exact, so all we can expect in the end is to gain a general impression.) The smallest value of the explosion that can result from preinitiation will be that given by a chain starting at $\alpha = 0$ and reached when the integral of $\alpha \cdot dt = 45$. The smallest possible yield resulting from preinitiation has been referred to as the "fizzle yield," Y_F . Letting α_F and t_F be the value of α and the supercritical time interval associated with the fizzle yield, we have $\frac{1}{2}\alpha_F \cdot t_F = 45$ or $(\alpha_F)^2/c = 90$. For the nominal situation we had $\alpha_0 \cdot t_0 = (\alpha_0)^2/c = 1,000$. From this, $(\alpha_F)^2/(\alpha_0)^2 = 90/1,000$ or $\alpha_F = 0.3 \cdot \alpha_0$. Using now $Y \sim \alpha^3$ gives $Y_F = 0.027 \cdot Y_0$. Roughly, then, for our hypothetical example the fizzle yield is in the range of a few percent of the nominal yield. Thus, if the nominal yield is 20 kilotons, the fizzle yield might be 0.5 kilotons.

Several observations can be made on the basis of the arguments used above. One is that, if one could achieve the same end state by using a more

rapid implosion, because of the decrease in t_0 , the value α_F would be a larger fraction of α_0 and Y_F would be a larger fraction of the nominal yield Y_0 . Similarly, if a specified design should be brought to a more effective final state (by having a larger α_0 as a consequence of increased compression, for example) though the fizzle yield might be a smaller fraction of the (increased) nominal yield, it would— at least on the basis of the approximations employed above— be larger in absolute value. And finally, if, as in a gun-type system (with an assembly velocity of approximately $3 \cdot 10^4$ cm sec⁻¹ rather than the $5 \cdot 10^5$ cm sec⁻¹ assumed above), the value of t_0 would be more than a factor of 10 times larger than that used above, the value of α_F/α_0 would be reduced by a factor larger than $\sqrt{10}$, and Y_F/Y_0 by a factor of more than 30. Along with this there would be the greatly increased probability of predetonation. This reveals the basis for the familiar statement that plutonium cannot be used in a gun-type assembly.

The arguments outlined above with regard to plutonium also apply to weapons-grade uranium. Apart from having a larger critical mass, the significant difference is that in 94 percent U-235 the inherent neutron source is smaller than that in weapons-grade plutonium by a factor of several thousand. The fizzle yield is indeed very small, but the neutron source is small enough that the probability of experiencing an early preinitiation in a design employing weapons-grade uranium is tolerable even in a gun-type assembly.

EFFECTS OF PREINITIATION ON YIELD DISTRIBUTION

One week after the first fission explosion on 16 July 1945, Robert Oppenheimer wrote to General Leslie R. Groves' deputy and described the expected performance of the Trinity device in combat,²

The possibility that the first combat plutonium Fat Man will give a less than optimal performance is about 12 percent. There is about six percent chance that the energy release will be under 5,000 tons, and about two percent chance that it will be under 1,000 tons. It should not be much less than 1,000 tons unless there is an actual malfunctioning of some of the components.

One week later General Groves wrote to the Chief of Staff,

There is a definite possibility, 12 percent rising to 20 percent, as we increase our rate of production at the Hanford Engineer Works, with the type of weapon tested that the blast will be smaller due to detonation in advance of the optimum time. But in any event, the explosion should be on the order of thousands of tons.

Evidently, both Oppenheimer and Groves were referring to what has been identified as the fizzle yield. They do not state a value for this yield, but in view of their statement that, "...it should not be much less than a thousand tons" it may be presumed that they were thinking of some value larger than one half of that and probably in the vicinity of 700 tons, or so. Because the nominal yield of this device was known by that time to be 20 kilotons, 700 tons would be 3.5 percent of that, and not inconsistent with what has already been said concerning fizzle yields of a device of this type. The Pu-240 content of the plutonium used in the Trinity device was not stated, but it must have been quite low, smaller than the six percent or so currently accepted as the definition of "weapons-grade" plutonium. The reactors that produced the plutonium used in July 1945 started operation only in December 1944, and only a low level of irradiation was achieved in the limited time available for irradiation. The principal effect of using reactor-grade plutonium in place of the high-purity plutonium available in summer 1945 would have been to increase the probabilities that the yield would fall short of the nominal yield, but it would not greatly change the actual value of the fizzle yield, which would always be equalled or exceeded.

With the improved data and the greatly improved calculational capabilities to provide more precise descriptions of the complex neutronic and hydrodynamic processes involved than those available in 1945, it is most likely that the particular numbers stated by Oppenheimer would require some revision. But that would not change the general pattern, so Oppenheimer's values will be adequate for our needs, which are of a purely qualitative nature.

Oppenheimer's breakdown of probabilities may be rephrased as follows: with the Trinity implosion assembly system and the grade of plutonium employed, the probability was 88 percent that the device would survive long enough without a chain being initiated that it would provide the nominal yield of 20 kilotons; about 94 percent that it would survive long enough that the yield would be greater than 5 kilotons (one quarter of the nominal yield); about 98 percent that it would provide a yield in excess of one kiloton. Only in two percent of all firings would the chain be initiated so early that the energy release would be between the fizzle yield and one kiloton. Changing only the neutron source changes these probabilities. For example, for a source n -times larger, the probability of surviving to produce the nominal yield would be only 0.88 to the n -th power, and so forth. In particular, for sources 10, 20, 30 or 40 times larger than the one at Trinity, the probabilities of reaching the yield levels indicated (and the fraction initiated very close to critical) would be as shown in table 3. The probabilities of reaching indicated yield levels for an assembly system twice as rapid as Trinity's is shown in table 4.

Table 3: Probability (based on Oppenheimer's letter) of achieving indicated yields in the assembly system used at Trinity with neutron sources of various sizes.

Neutron source (multiple of Trinity)	Yield			
	Nominal (20 kilotons)	above 5 kt	above 1 kt	fizzle to 1 kt
Trinity	.88	.94	.98	.02
10 X	.28	.54	.82	.18
20 X	.08	.29	.67	.33
30 X	.02	.16	.55	.45
40 X	.006	.08	.45	.55

Table 4: Probability (based on Oppenheimer's letter) of achieving indicated yields in an assembly system twice as rapid as Trinity with neutron sources of various sizes.

Neutron source (multiple of Trinity)	Yield			
	Nominal (20 kilotons)	above 5 kt	above 1 kt	fizzle to 1 kt
Trinity	.94	.97	.99	.01
10 X	.54	.74	.90	.10
20 X	.28	.54	.82	.18
30 X	.16	.40	.74	.26
40 X	.08	.30	.67	.33

The largest neutron source in table 3 is probably larger than that in the most heavily exposed plutonium presently considered (see table 5). The substitution of a somewhat larger mass of reactor-grade plutonium for the high-grade plutonium used in the Trinity device would affect both the nominal yield and the fizzle yield. However, the general pattern pictured above would continue to apply: in assembly systems similar to Trinity's, a mass of reactor plutonium of any grade could have a nominal yield of the order of tens of kilotons and an associated fizzle yield of a few percent of the nominal yield—which is to say several, or even many, hundreds of tons. As the neutron source is increased from a low level to a very high level the distribution of yields realized would change from one in which the nominal yield is the typical and very

Table 5: Selected properties of various grades of plutonium.

Grade	Spontaneous fission neutrons (<i>gm-sec</i>) ⁻¹	Decay heat watts <i>kg</i> ⁻¹
Super-grade	20	2.0
Weapons-grade	66	2.3
Reactor-grade	360	10.5
MOX-grade	570	13.7

severe preinitiation is rare, to one in which the nominal yield is rare (though never completely excluded) and typical yields lie in a band from one to a few times larger than the fizzle yield. Though much smaller than the nominal yield (by a factor of about 20 in the particular case considered—the so-called “primitive” Trinity-style device) these near-fizzle yields would still constitute severely damaging explosions. Very heavy damage and acute hazard from the blast, thermal, and prompt radiation effects, which extended out to a radius of about a mile in the case of the weapons used in Japan, would, for these “small” yields, extend out “only” to a radius of one-third or one-half a mile.

HEAT

Associated with the wide range of α activity among the materials listed in table 5 there will be a wide range in the heat and radiation emitted by these materials. Weapons-grade material (which is handled routinely) generates about 2.5 watts per kilogram, whereas the reactor-grade material would generate more than 10.5 watts per kilogram. As Gerhard Locke has recently emphasized, a crude nuclear explosive containing perhaps eight kilograms of reactor-grade plutonium would put out nearly 100 watts of heat—much more than the eight watts emitted from the approximately three kilograms of weapons-grade plutonium he suggests would be in a modern nuclear warhead.³

Since the high-explosive (HE) around the plutonium core would have insulating properties only a few times poorer than wood (about 0.4 watts *m-°C*⁻¹)⁴, only 10 centimeters of HE could result in an equilibrium temperature of the core of about 190°C.⁵ Apparently, the breakdown rate of many types of HE begins to become significant above about 100°C.

As to emitted radiation, Johan Swahn of the Technical Peace Research Group of Chalmers University in Goteborg, Sweden has developed data⁶ indi-

cating that the surface dose exposure rate of material such as the reactor-grade plutonium of table 5 is about six times larger (and MOX-grade over eight times larger) than that from the weapons-grade material which, again, is handled routinely.

The design of a crude nuclear explosive using reactor-grade plutonium will have to account for the extra heat generation and radiation exposure, but provisions can certainly be devised to cope with these features. For example, since the thermal conductivity of aluminum is almost 1,000 times greater than that of HE, a thermal bridge with a total cross-section at the surface of the core of only about one cm^2 could halve the temperature increase induced by the reactor-grade plutonium.

CONCLUSIONS

- ◆ Reactor-grade plutonium with any level of irradiation is a potentially explosive material.
- ◆ The difficulties of developing an effective design of the most straightforward type are not appreciably greater with reactor-grade plutonium than those that have to be met for the use of weapons-grade plutonium.
- ◆ The hazards of handling reactor-grade plutonium, though somewhat greater than those associated with weapons-grade plutonium, are of the same type and can be met by applying the same precautions. This, at least, would be the case when fabricating only a modest number of devices. For a project requiring an assembly line type of operation, more provisions for remote handling procedures for some stages of the work might be required than would be necessary for handling weapons-grade material or for handling a limited number of items.
- ◆ The need for safeguards to protect against the diversion and misuse of separated plutonium applies essentially equally to all grades of plutonium.

NOTES AND REFERENCES

1. Robert Serber, *The Los Alamos Primer: The First Lectures on How to Build an Atomic Bomb* (Berkeley, California: University of California Press, 1992). (Notes on lectures given by Serber in 1943 at the start of the Los Alamos Laboratory, declassified circa 1965.)
2. Quoted by Albert Wohlstetter in *Foreign Policy* 25, winter 1976–1977, p. 160.
3. Gerhard Locke, "Why Reactor-Grade Plutonium is No Nuclear Explosive Suitable

for Military Devices," prepared for Workshop on the Disposition of Plutonium, Bonn, 13–16 June 1992.

4. According to Egbert Kankeleit, Christian Küppers and Ulrich Imkeller, *Bericht zur Waffentauglichkeit von Reaktorplutonium* (IANUS Institute für Kernphysik, Technische Hochschule Darmstadt).

5. Assuming an outer radius for the core of seven centimeters and an ambient temperature of 20°C.

6. Johan Swahn, "The Long-Range Nuclear Explosives Predicament" (Göteborg: Technical Peace Research Group, Institute of Physical Research Theory, 1992).

Appendix: Probabilities of Different Yields

Frank von Hippel^a and Edwin Lyman^a

The calculations in this appendix are based on Mark's simplified model of the behavior of an implosion design.

This model assumes a linear growth of the neutron multiplication rate from zero at time $t = 0$ to unity at the time of maximum supercriticality, $t = t_0$

$$(k - 1) = \frac{t}{t_0}$$

The exponential time constant for the neutron chain reaction is then

$$\alpha \equiv \frac{(k - 1)}{\tau} = \frac{t}{(t_0 \tau)}$$

where τ is the average time between neutron generations.

Mark's criterion for predetonation is that the chain-reaction be initiated at a time t_i early enough so that approximately e^{45} fissions have occurred before maximum criticality is achieved, i.e.,

$$\int_{t_i}^{t_f} \alpha(t) dt = \frac{1}{2} \left[\frac{1}{(t_0 \tau)} \right] [(t_f)^2 - (t_i)^2] = 45$$

when $t_f < t_0$. Solving for t_f gives

$$t_f = [(t_i)^2 + 90t_0\tau]^{1/2} \quad (\text{A-1})$$

The design yield, Y_0 , will be achieved when $t_f \geq t_0$ or

$$Y = Y_0, \text{ when } t_i \geq t_0 \left(1 - \frac{90\tau}{t_0} \right)^{1/2} \equiv t_i^{\text{crit}} \quad (\text{A-2})$$

For Mark's values, $t_0 = 10^{-5}$ and $\tau = 10^{-8}$ seconds, this corresponds to $t_i \geq 0.954 \cdot 10^{-5}$ seconds.

Mark also uses the approximation, derived in the *Los Alamos Primer*,¹ relating the reduced predonation yield Y to the design yield Y_0

$$Y = [(k_f - 1)^3] Y_0 = \left(\frac{t_f}{t_0} \right)^3 Y_0, \quad t_f < t_0 \quad (\text{A-3})$$

a. Center for Energy and Environmental Studies, Princeton University, Princeton, New Jersey.

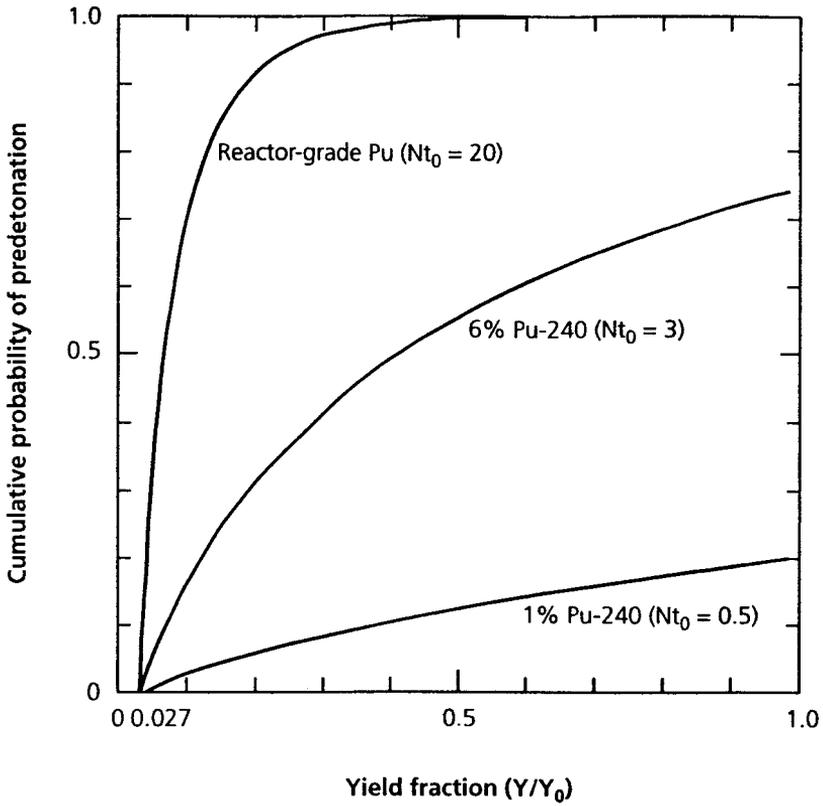


Figure A-1: $P(Y/Y_0 < 1)$ as a function of y/y_0 for plutonium cores with $N = 0.5, 3,$ and $20 \cdot 10^5$ spontaneous fissions per second for $t_0 = 10^{-5}$ and $\tau = 10^{-8}$ seconds.

where

$$k_f \equiv k(t_f) = \frac{t_f}{t_0}$$

From equation A-1 the minimum value of t_f is given by

$$(t_f)_{\min} = (90t_0\tau)^{1/2}$$

Therefore, from equation A-3, the minimum value of Y/Y_0 is

$$\left(\frac{Y}{Y_0}\right)_{\min} = \left(\frac{90\tau}{t_0}\right)^{3/2} \tag{A-4}$$

Spontaneous fissions in the plutonium in the warhead generate neutrons at a rate of N per second. For six kilograms of weapon-grade or reactor-grade plutonium, N is approximately $3 \cdot 10^5$ or $20 \cdot 10^5 \text{ sec}^{-1}$, respectively. We also consider below a case with $N = 0.5 \cdot 10^5 \text{ sec}^{-1}$ (one percent Pu-240), which we find produces approximately the probabilities of reduced yields for the Trinity test estimated in Oppenheimer's letter to Groves.

The expected value that one of the neutrons will start a chain reaction is $(k - 1)$. The probability P of a chain reaction having been initiated by time T is therefore²

$$P(t < T) = 1 - \exp[-NT(k - 1)_{\text{av}}] = 1 - \exp\left[-\frac{1}{2}NT\left(\frac{T}{t_0}\right)\right] \quad (\text{A-5})$$

where we have used the fact that, since $(k - 1)$ increases linearly

$$(k - 1)_{\text{av}} = \frac{1}{2}(k - 1) = \frac{1}{2}\left(\frac{T}{t_0}\right)$$

From equations A-2 and A-5, the probability of an explosion with full yield is then

$$1 - P(t < t_i^{\text{crit}}) = \exp\left[-\frac{1}{2}N(t_0 - 90\tau)\right] \quad (\text{A-6})$$

For $t_0 = 10^{-5}$ seconds, $\tau = 10^{-8}$ seconds, and $N = 0.5, 3.0$ and $20 \cdot 10^5 \text{ sec}^{-1}$ the probabilities that the neutron chain reaction will *not* start before full yield is guaranteed are then calculated as 79.7, 25.5 and 0.0112 percent, respectively.

The differential probability of a chain reaction starting at an earlier time t_i is

$$\frac{dP}{dt_i} = N\left(\frac{t_i}{t_0}\right) \exp\left[-\frac{1}{2}N\left(\frac{t_i^2}{t_0}\right)\right]$$

The probability density of a reduced yield (dP/dY) is then given by the chain rule as

$$\begin{aligned} \frac{dP}{dY} &= \left(\frac{dP}{dt_i}\right)\left(\frac{dt_i}{dY}\right) \\ &= \left(\frac{dP}{dt_i}\right)\left(\frac{dt_i}{dt_f}\right)\left[\frac{dt_f}{d(k_f - 1)}\right]\left[\frac{d(k_f - 1)}{dY}\right] \\ &= \left(\frac{dP}{dt_i}\right)\left(\frac{t_f}{t_i}\right)(t_0) \frac{1}{3}\left[\left(\frac{t_0}{t_f}\right)^2\right]\left(\frac{1}{Y_0}\right) \\ &= \frac{1}{3}\left(\frac{1}{Y_0}\right)N\left(\frac{t_0^2}{t_f}\right) \exp\left[-\frac{1}{2}Nt_0\left(\frac{t_i}{t_0}\right)^2\right] \end{aligned}$$

* The Trinity device reportedly contained 6.1 kilograms of plutonium. (General Leslie R. Groves, Memorandum to the Secretary of War, 18 July 1945, reprinted as Appendix P in Martin Sherwin, *A World Destroyed* [New York: Alfred A. Knopf, 1975].)

Defining

$$x \equiv \frac{Y}{Y_0}$$

and substituting for t_i from equation A-1 and for t_f from equation A-3 we get

$$\frac{dP}{dx} = \left[\frac{1}{3} N t_0 \exp(45N\tau) \right] x^{-1/3} \exp \left[-\frac{1}{2} N t_0 x^{2/3} \right]$$

Finally, integrating from

$$x_{\min} = \left(\frac{90\tau}{t_0} \right)^{3/2}$$

(see equation A-4) to x gives

$$P \left(\frac{Y}{Y_0} < x \right) = \int_{x_{\min}}^x dx' \left(\frac{dP}{dx'} \right) = 1 - \exp \left[-\frac{1}{2} N t_0 x^{2/3} + 45N\tau \right]$$

When $x = x_{\max} = 1$, we have $P(Y/Y_0 < 1)$ or the same total probability of predetonation with reduced yield that could have been calculated above from equation A-6 and

$$P \left(\frac{Y}{Y_0} < 1 \right) = 1 - P(Y = Y_0).$$

NOTES AND REFERENCES

1. *The Los Alamos Primer*, p. 42.
2. *The Los Alamos Primer*, p. 48.